20. $\cot(-x)\tan(-x)$

21. $\sin^2(-x) + \cos^2(-x)$

22. $\sec^2(-x) - \tan^2 x$

In Exercises 23–26, simplify the expression to either a constant or a basic trigonometric function. Support your result graphically.

23. $\dfrac{\tan(\pi/2 - x)\csc x}{\csc^2 x}$

24. $\dfrac{1 + \tan x}{1 + \cot x}$

25. $(\sec^2 x + \csc^2 x) - (\tan^2 x + \cot^2 x)$

26. $\dfrac{\sec^2 u - \tan^2 u}{\cos^2 v + \sin^2 v}$

In Exercises 27–32, use the basic identities to change the expression to one involving only sines and cosines. Then simplify to a basic trigonometric function.

27. $(\sin x)(\tan x + \cot x)$

28. $\sin\theta - \tan\theta\cos\theta + \cos(\pi/2 - \theta)$

29. $\sin x \cos x \tan x \sec x \csc x$

30. $\dfrac{(\sec y - \tan y)(\sec y + \tan y)}{\sec y}$

31. $\dfrac{\tan x}{\csc^2 x} + \dfrac{\tan x}{\sec^2 x}$

32. $\dfrac{\sec^2 x \csc x}{\sec^2 x + \csc^2 x}$

In Exercises 33–38, combine the fractions and simplify to a multiple of a power of a basic trigonometric function (e.g., $3\tan^2 x$).

33. $\dfrac{1}{\sin^2 x} + \dfrac{\sec^2 x}{\tan^2 x}$

34. $\dfrac{1}{1 - \sin x} + \dfrac{1}{1 + \sin x}$

35. $\dfrac{\sin x}{\cot^2 x} - \dfrac{\sin x}{\cos^2 x}$

36. $\dfrac{1}{\sec x - 1} - \dfrac{1}{\sec x + 1}$

37. $\dfrac{\sec x}{\sin x} - \dfrac{\sin x}{\cos x}$

38. $\dfrac{\sin x}{1 - \cos x} + \dfrac{1 - \cos x}{\sin x}$

In Exercises 39–46, write each expression in factored form as an algebraic expression of a single trigonometric function (e.g., $(2\sin x + 3)(\sin x - 1)$).

39. $\cos^2 x + 2\cos x + 1$

40. $1 - 2\sin x + \sin^2 x$

41. $1 - 2\sin x + (1 - \cos^2 x)$

42. $\sin x - \cos^2 x - 1$

43. $\cos x - 2\sin^2 x + 1$

44. $\sin^2 x + \dfrac{2}{\csc x} + 1$

45. $4\tan^2 x - \dfrac{4}{\cot x} + \sin x \csc x$

46. $\sec^2 x - \sec x + \tan^2 x$

In Exercises 47–50, write each expression as an algebraic expression of a single trigonometric function (e.g., $2\sin x + 3$).

47. $\dfrac{1 - \sin^2 x}{1 + \sin x}$

48. $\dfrac{\tan^2 \alpha - 1}{1 + \tan \alpha}$

49. $\dfrac{\sin^2 x}{1 + \cos x}$

50. $\dfrac{\tan^2 x}{\sec x + 1}$

In Exercises 51–56, find all solutions to the equation in the interval $[0, 2\pi)$. You do not need a calculator.

51. $2\cos x \sin x - \cos x = 0$

52. $\sqrt{2}\tan x \cos x - \tan x = 0$

53. $\tan x \sin^2 x = \tan x$

54. $\sin x \tan^2 x = \sin x$

55. $\tan^2 x = 3$

56. $2\sin^2 x = 1$

In Exercises 57–62, find all solutions to the equation. You do not need a calculator.

57. $4\cos^2 x - 4\cos x + 1 = 0$

58. $2\sin^2 x + 3\sin x + 1 = 0$

59. $\sin^2\theta - 2\sin\theta = 0$

60. $3\sin t = 2\cos^2 t$

61. $\cos(\sin x) = 1$

62. $2\sin^2 x + 3\sin x = 2$

In Exercises 63–68, find all solutions to the trigonometric equation, using a calculator where needed.

63. $\cos x = 0.37$

64. $\cos x = 0.75$

65. $\sin x = 0.30$

66. $\tan x = 5$

67. $\cos^2 x = 0.4$

68. $\sin^2 x = 0.4$

In Exercises 69–74, make the suggested trigonometric substitution, and then use Pythagorean identities to write the resulting function as a multiple of a basic trigonometric function.

69. $\sqrt{1 - x^2}$, $x = \cos\theta$

70. $\sqrt{x^2 + 1}$, $x = \tan\theta$

71. $\sqrt{x^2 - 9}$, $x = 3\sec\theta$

72. $\sqrt{36 - x^2}$, $x = 6\sin\theta$

73. $\sqrt{x^2 + 81}$, $x = 9\tan\theta$

74. $\sqrt{x^2 - 100}$, $x = 10\sec\theta$

Standardized Test Questions

75. True or False If $\sec(x - \pi/2) = 34$, then $\csc x = 34$. Justify your answer.

76. True or False The domain of validity for the identity $\sin\theta = \tan\theta\cos\theta$ is the set of all real numbers. Justify your answer.

You should answer these questions without using a calculator.

77. Multiple Choice Which of the following could *not* be set equal to $\sin x$ as an identity?

(A) $\cos\left(\dfrac{\pi}{2} - x\right)$ **(B)** $\cos\left(x - \dfrac{\pi}{2}\right)$

(C) $\sqrt{1 - \cos^2 x}$ **(D)** $\tan x \sec x$

(E) $-\sin(-x)$

78. Multiple Choice Exactly four of the six basic trigonometric functions are

(A) odd. **(B)** even.

(C) periodic. **(D)** continuous.

(E) bounded.

79. Multiple Choice A simpler expression for $(\sec\theta + 1)(\sec\theta - 1)$ is

(A) $\sin^2\theta$. **(B)** $\cos^2\theta$.

(C) $\tan^2\theta$. **(D)** $\cot^2\theta$.

(E) $\sec^2\theta$.

80. Multiple Choice How many numbers between 0 and 2π solve the equation $3\cos^2 x + \cos x = 2$?

(A) none **(B)** one

(C) two **(D)** three

(E) four

Explorations

81. Write all six basic trigonometric functions entirely in terms of $\sin x$.

82. Write all six basic trigonometric functions entirely in terms of $\cos x$.

83. Writing to Learn Graph the functions $y = \sin^2 x$ and $y = -\cos^2 x$ in the standard trigonometric viewing window. Describe the apparent relationship between these two graphs and verify it with a trigonometric identity.

84. Writing to Learn Graph the functions $y = \sec^2 x$ and $y = \tan^2 x$ in the standard trigonometric viewing window. Describe the apparent relationship between these two graphs and verify it with a trigonometric identity.

85. Orbit of the Moon Because its orbit is elliptical, the distance from the Moon to the Earth in miles (measured from the center of the Moon to the center of the Earth) varies periodically. On Monday, January 18, 2002, the Moon was at its apogee (farthest from the earth). The distance of the Moon from the Earth each Friday from January 23 to March 27 are recorded in Table 5.1.

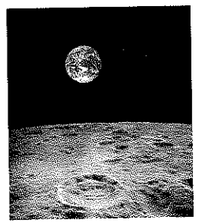

Table 5.1 Distance from Earth to Moon

Date	Day	Distance
Jan 23	0	251,966
Jan 30	7	238,344
Feb 6	14	225,784
Feb 13	21	240,385
Feb 20	28	251,807
Feb 27	35	236,315
Mar 6	42	226,101
Mar 13	49	242,390
Mar 20	56	251,333
Mar 27	63	234,347

Source: The World Almanac and Book of Facts, 2005.

(a) Draw a scatter plot of the data, using "day" as x and "distance" as y.

(b) Use your calculator to do a sine regression of y on x. Find the equation of the best-fit sine curve and superimpose its graph on the scatter plot.

(c) What is the approximate number of days from apogee to apogee? Interpret this number in terms of the orbit of the moon.

(d) Approximately how far is the Moon from the Earth at perigee (closest distance)?

(e) Since the data begin at apogee, perhaps a cosine curve would be a more appropriate model. Use the sine curve in part (b) and a cofunction identity to write a cosine curve that fits the data.

86. Group Activity Divide your class into six groups, each assigned to one of the basic trigonometric functions. With your group, construct a list of five different expressions that can be simplified to your assigned function. When you are done, exchange lists with your "cofunction" group to check one another for accuracy.

Extending the Ideas

87. Prove that $\sin^4\theta - \cos^4\theta = \sin^2\theta - \cos^2\theta$.

88. Find all values of k that result in $\sin^2 x + 1 = k\sin x$ having an infinite solution set.

89. Use the cofunction identities and odd-even identities to prove that $\sin(\pi - x) = \sin x$.
[*Hint*: $\sin(\pi - x) = \sin(\pi/2 - (x - \pi/2))$.]

90. Use the cofunction identities and odd-even identities to prove that $\cos(\pi - x) = -\cos x$.
[*Hint*: $\cos(\pi - x) = \cos(\pi/2 - (x - \pi/2))$.]

91. Use the identity in Exercise 89 to prove that in any $\triangle ABC$, $\sin(A + B) = \sin C$.

92. Use the identities in Exercises 89 and 90 to find an identity for simplifying $\tan(\pi - x)$.